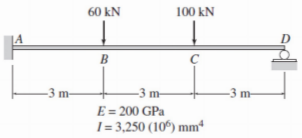
นางสาวโชติกาญจน์ สังข์เปรม 6306100035 SEC001

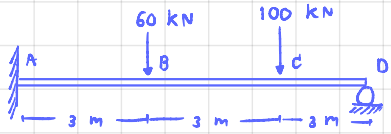
ข้อ 1 ให้วาด Shear Force Diagram (SFD) และ Bending Moment Diagram (BMD) ของโจทย์ที่กำหนด
โดยเลือกใช้วิธีใดวิธีหนึ่งระหว่าง Virtual Work หรือ Conjugate Beam



D.I. = r-(e+c) = 4-3 = 1

ให้ D_y เป็น Redundant

จะได้ว่า



=

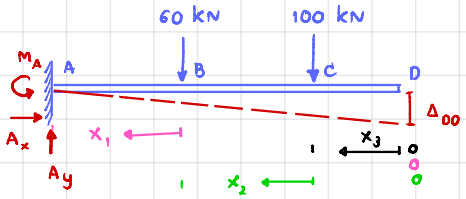
โครงสร้างหลัก

+

โครงสร้างรับแรงส่วนเกิน

จะได้ $\Delta_D = 0 = \Delta_{DD} + f_{DD} D_y$

① หา Δ_{DD}



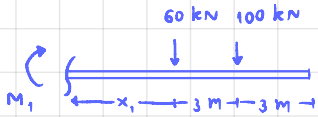
$[\Sigma F_y = 0]$; $A_y = 160$ kN

$[\Sigma F_x = 0]$; $A_x = 0$ kN

$[\Sigma M_A = 0]$ ⊕ ; $-60(3) - 100(6) + M_A = 0$

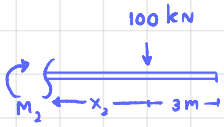
$M_A = 780$ kN·m

Actual

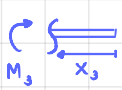
① 

$M_1 = -60x_1 - 100(x_1 + 3)$

$M_1 = -160x_1 - 300$

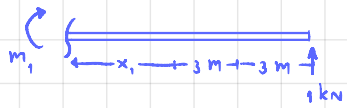
② 

$M_2 = -100x_2$

③ 

$M_3 = 0$

Vertual ;

① 

$m_1 = 1(x_1 + 6)$

② 

$m_2 = 1(x_2 + 3)$

③ 

$m_3 = 1(x_3)$

จาก $1(\Delta_{DD}) = \int_0^L \frac{Mm}{EI}\,dx$

$\Delta_{DD} = \frac{1}{EI}\left[\int_0^3 (-160x - 300)(x+6)\,dx + \int_0^3 (-100x)(x+3)\,dx\right]$

$\Delta_{DD} = \frac{1}{EI}(-12510 - 2250) = \frac{-14760}{EI}$

② การหา f_{DD} สามารถอ้างอิงช่วง section ทั้ง Actual และ Vertual ตามโครงสร้างหลักได้

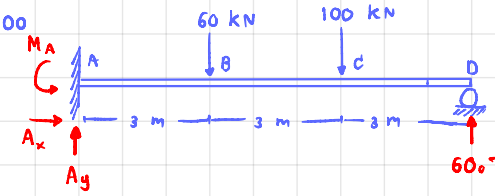
$1 \cdot f_{BB} = \frac{1}{EI}\left[\int_0^3 (x+6)^2\,dx + \int_0^3 (x+3)^2\,dx + \int_0^3 x^2\,dx\right]$

$f_{BB} = \frac{243}{EI}$

จาก $0 = \Delta_{DD} + f_{DD} D_y$

จะได้ $0 = \frac{-14760}{EI} + \frac{243}{EI} D_y$

$D_y = 60.74$ kN



$[\Sigma F_y = 0]$;

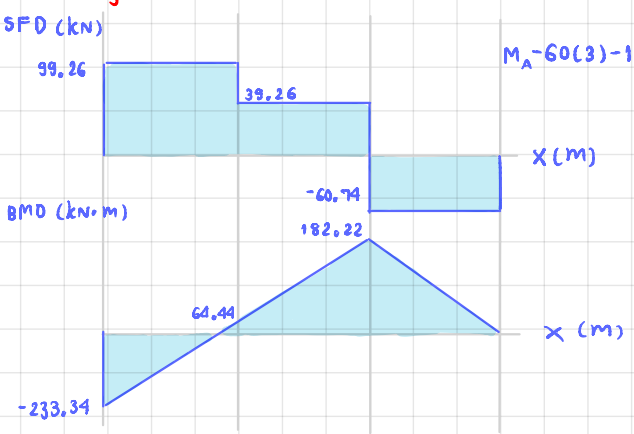
$A_y = 60 + 100 - 60.74$

$A_y = 99.26$ kN

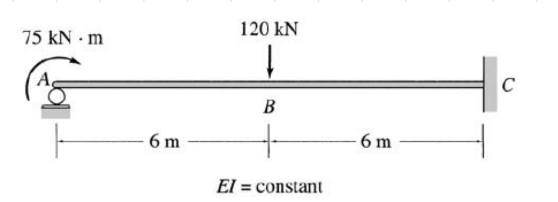
$[\Sigma M_A = 0]$ ⊕ ;

$M_A - 60(3) - 100(6) + 60.74(9) = 0$

$M_A = 233.34$ kN·m



นางสาวโชติกาญจน์ สังข์เปรม 6306100035 SEC001



EI = constant

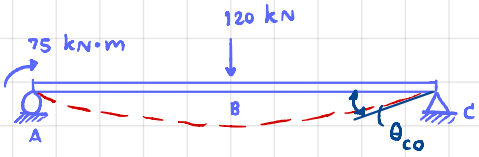
D.I. = r-(e+c) = 4-3 = 1

ให้ M_c เป็น Redundant

จะได้ว่า

=

โครงสร้างหลัก

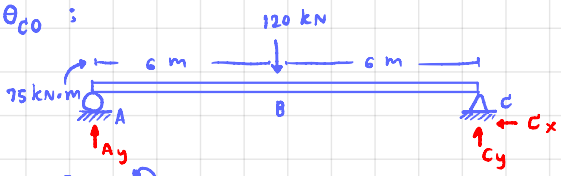


+

โครงสร้างรับแรงส่วนเกิน

จะได้ $\theta_c = 0 = \theta_{co} + f_{cc} M_c$

① หา θ_{co} :

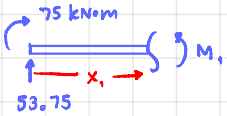


$[\Sigma M_A = 0]$; ⊕ $-120(6) - 75 + C_y(12) = 0$

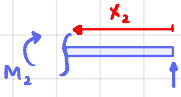
$C_y = 66.25$ kN

$[\Sigma F_y = 0]$; $A_y = 120 - 66.25 = 53.75$ kN

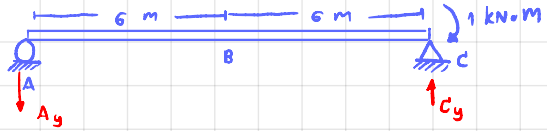
Actual ;

① 

$M_1 = 75 + 53.75x_1$

② 

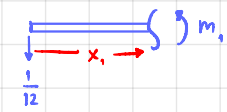
$M_2 = 66.25x_2$

Vertual ; 

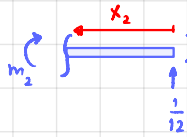
$[\Sigma M_C = 0]$ ⊕ ; $A_y(12) - 1 = 0$

$A_y = \frac{1}{12}$ kN

$[\Sigma F_y = 0]$; $C_y = \frac{1}{12}$ kN

① 

$m_1 = -\frac{x_1}{12}$

② 

$m_2 = -1 + \frac{x_2}{12}$

จาก $1(\theta_{co}) = \int_0^L \frac{mM}{EI} dx$

$$\theta_{co} = \frac{1}{EI}\left[\int_0^6 (75 + 53.75x)\left(-\frac{x}{12}\right) dx + \int_0^6 (66.25x)\left(-1 + \frac{x}{12}\right) dx\right]$$

$$\theta_{co} = \frac{-1230}{EI}$$

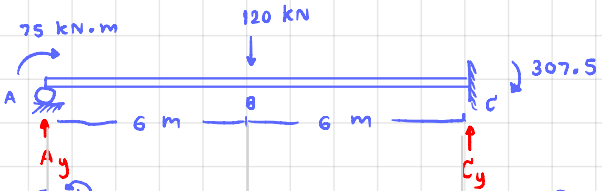
② การหา f_{cc} สามารถอ้างอิงช่วง section ทั้ง Actual และ Vertual ตามโครงสร้างหลักได้

$$f_{cc} = \frac{1}{EI}\left[\int_0^6 \left(-\frac{x}{12}\right)^2 dx + \int_0^6 \left(-1 + \frac{x}{12}\right)^2 dx\right] = \frac{4}{EI}$$

จาก $0 = \theta_{co} + f_{cc} M_c$

จะได้ $0 = \frac{-1230}{EI} + \frac{4}{EI} M_c$

$M_c = 307.5$ kN·m (ตามเข็ม)

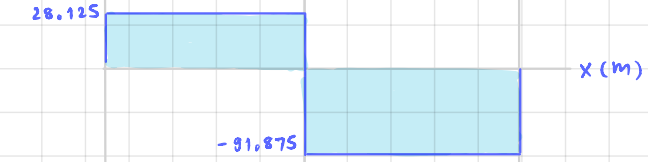


$[\Sigma M_C = 0]$ ⊕ ; $-75 - 307.5 + 120(6) - A_y(12) = 0$

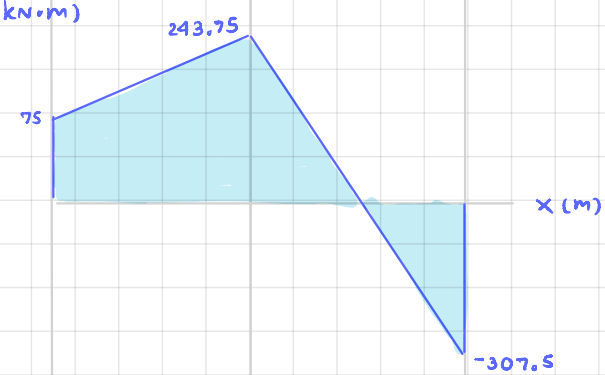
$A_y = 28.125$ kN

$[\Sigma F_y = 0]$; $C_y = 120 - 28.125 = 91.875$ kN

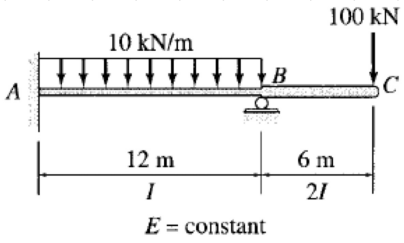
SFD (kN)



BMD (kN·m)



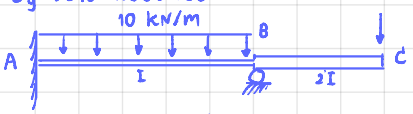
นางสาวโชติกาญจน์ สังข์เปรม 630610035 SEC001



$D.I_0 = r-(e+c) = 4-3 = 1$

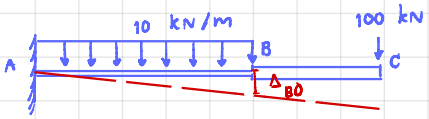
ให้ B_y เป็น Redundant

จะได้ว่า



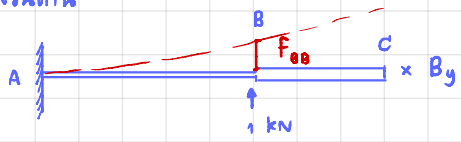
=

โครงสร้างหลัก



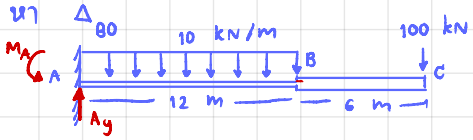
+

โครงสร้างรับแรงส่วนเกิน



จะได้ $\Delta_B = 0 = \Delta_{B0} + f_{BB} B_y$

① หา Δ_{B0}



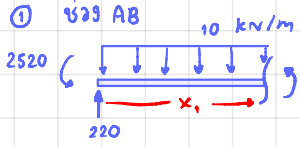
$[\Sigma F_y = 0]$; $A_y = 10(12)+100 = 220$ kN

$\circlearrowleft [\Sigma M_A = 0]$; $M_A - 10(12)(6) - 100(18) = 0$

$M_A = 2520$ kN·m

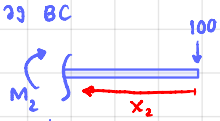
Actual ;

① ช่วง AB



$M_1 = 220x_1 - 2520 - 5x_1^2$

② ช่วง BC



$M_2 = -100x_2$

Vertual ;

$[\Sigma F_y = 0]$; $A_y = 1$ kN

$\circlearrowleft [\Sigma M_A = 0]$; $M_A = 12$ kN·m

① ช่วง AB

$m_1 = 12 - x_1$

② ช่วง BC

$m_2 = 0$

จาก $1(\Delta_{B0}) = \int_0^L \frac{Mm}{EI}\,dx$

$\Delta_{B0} = \frac{1}{EI}\int_0^{12}(220x - 2520 - 5x^2)(12-x)\,dx$

$\Delta_{B0} = \frac{-126720}{EI}$

② การหา f_{BB} สามารถอ้างอิงช่วง section ทั้ง Actual และ Vertual ตามโครงสร้างหลักได้

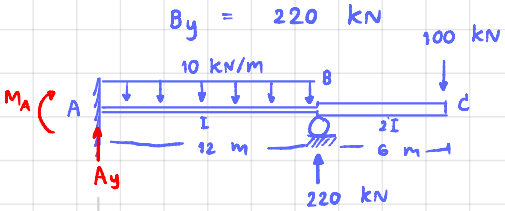
$f_{BB} = \frac{1}{EI}\int_0^{12}(12-x)^2\,dx$

$f_{BB} = \frac{576}{EI}$

จาก $0 = \Delta_{B0} + f_{BB} B_y$

$0 = \frac{-126720}{EI} + \frac{576}{EI} B_y$

$B_y = 220$ kN

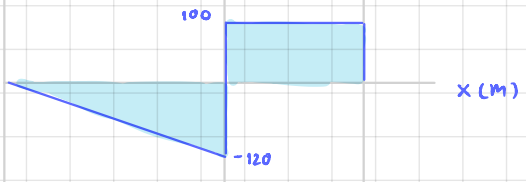


$[\Sigma F_y = 0]$; $A_y = 10(12) + 100 - 220 = 0$

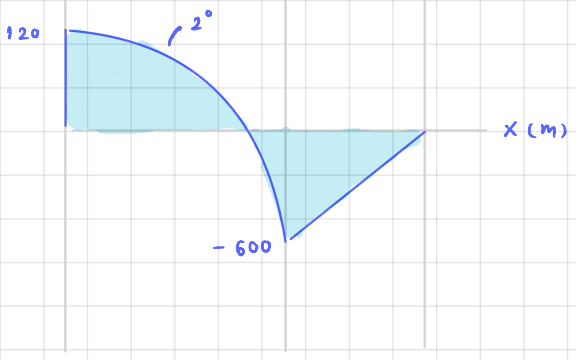
$\circlearrowleft [\Sigma M_A = 0]$; $M_A = -10(12)(6) + 220(12) - 100(18)$

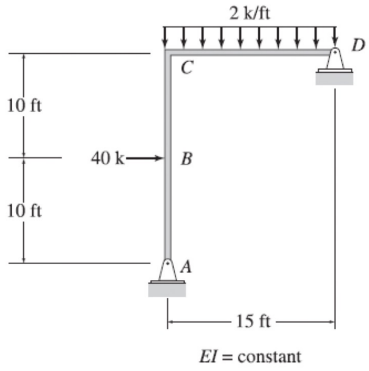
$M_A = 120$ kN·m

SFD (kN)



BMD (kN·m)



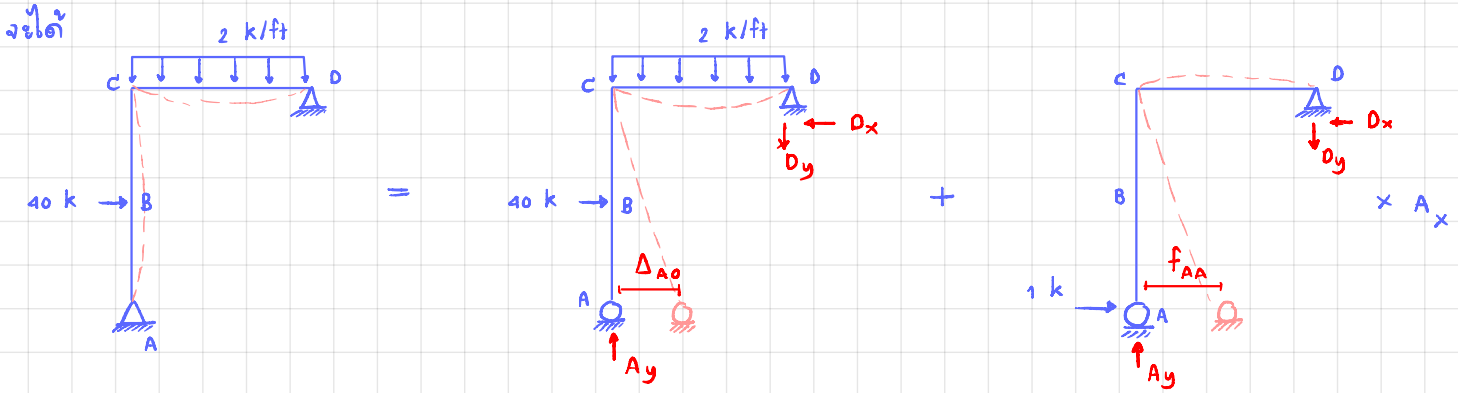


D.I. = (3m+r) - (3j+c)

D.I. = (6+4) - (9) = 1

ให้ A_x เป็น Redundant

จะได้



โครงสร้างหลัก

$[\Sigma M_D = 0]$ ↻+ ;

$-A_y(15) + 40(10) + 30(7.5) = 0$

$A_y = 41.67$

$[\Sigma F_y = 0]$; $D_y = 41.67 - 2(15) = 11.67$ k

$D_x = 40$ k

โครงสร้างส่วนเกิน

$[\Sigma M_D = 0]$ ↻+ ;

$1(20) - A_y(15) = 0$

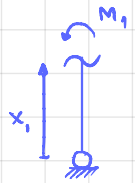
$A_y = 1.33$ k

$D_y = 1.33$ k $D_x = 1$ k

จะได้ $\Delta_A = 0 = \Delta_{AO} + f_{AA} A_x$

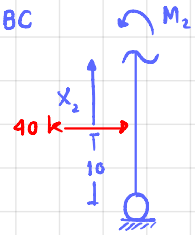
① หา Δ_{AO}

Actual ; ช่วง AB



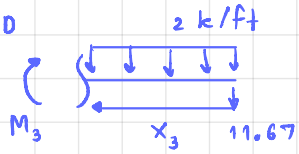
$M_1 = 0$

ช่วง BC



$M_2 = -40x_2$

ช่วง CD

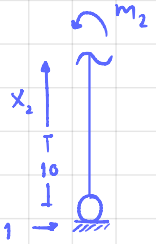


$M_3 = -x_3^2 - 11.67x_3$

Vertual ; ช่วง AB

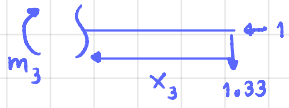
$m_1 = -x_1$

ช่วง BC



$m_2 = -(x_2 + 10)$

ช่วง CD



$m_3 = -1.33x_3$

จาก $\Delta_{AO} = \int_0^L \frac{Mm}{EI} dx$

$$\Delta_{AO} = \frac{1}{EI}\left[\int_0^{10} (40x)(x+10)\,dx + \int_0^{15} (-x^2 - 11.67x)(-1.33x)\,dx\right]$$

$$\Delta_{AO} = \frac{67627.38}{EI}$$

② การหา f_{AA} สามารถอ้างอิงช่วง section ทั้ง Actual และ Vertual ตามโครงสร้างหลักได้

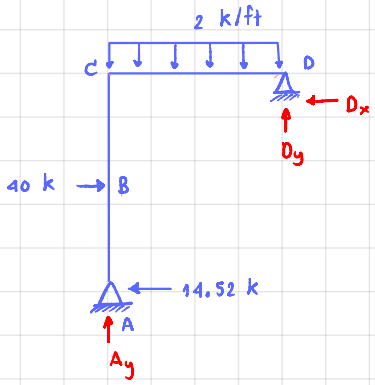
$$f_{AA} = \frac{1}{EI}\left[\int_0^{10} x^2\,dx + \int_0^{10} (x+10)^2\,dx + \int_0^{15} (1.33x)^2\,dx\right]$$

$$f_{AA} = \frac{4656.68}{EI}$$

จาก $\quad 0 = \Delta_{A0} + f_{AA} A_x$

$$0 = \frac{67627.38}{EI} + \frac{4656.68}{EI} A_x$$

$$A_x = 14.52 \text{ k}$$



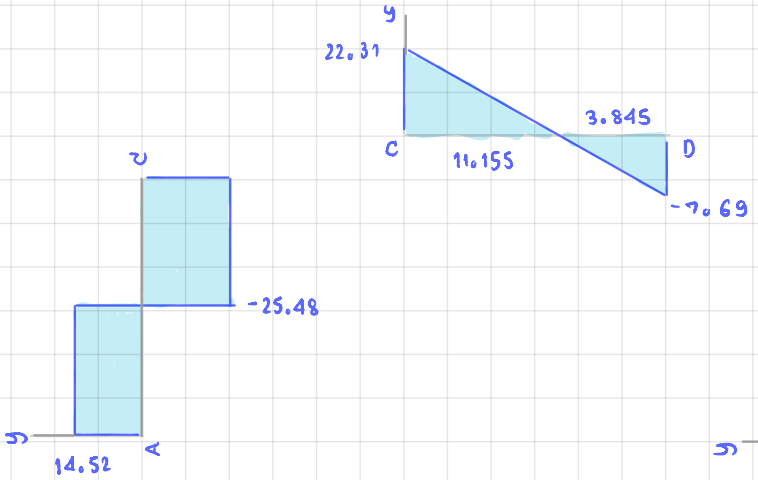
$[\Sigma F_x = 0]$; $D_x = 40 - 14.52 = 25.48$ k

$[\Sigma M_A = 0]$; $-2(15)(7.5) + D_y(15) + 25.48(20) - 40(10) = 0$

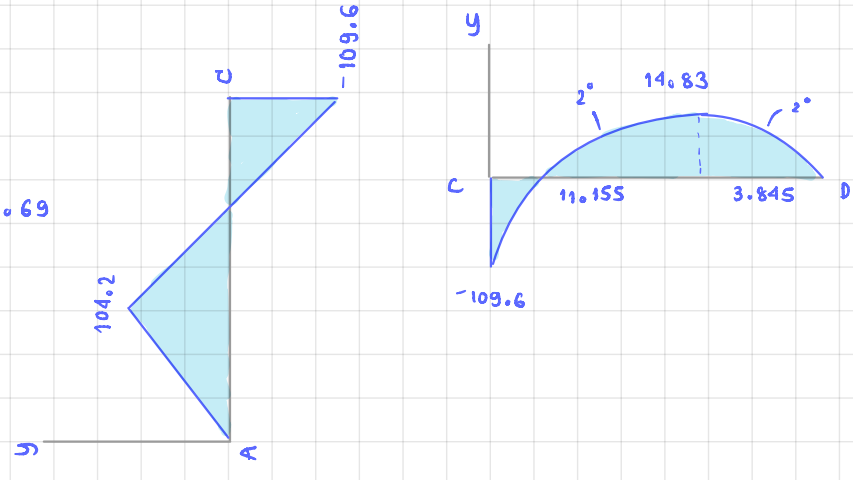
$D_y = 7.69$ k

$[\Sigma F_y = 0]$; $A_y = 30 - 7.69 = 22.31$ k

SFD (k)



BMD (k·ft)



Δ คล้าย ; $\quad \frac{22.31}{x} = \frac{7.69}{15-x}$

$x = 11.155$

ข้อ 2 ให้หาแรงภายในของชิ้นส่วนในโครงถัก

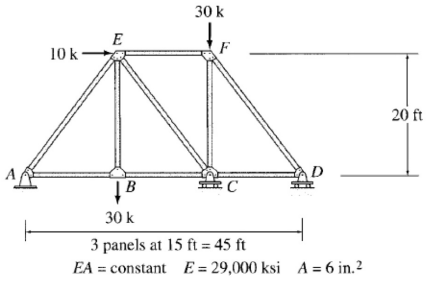


FIG. P13.27, P13.52

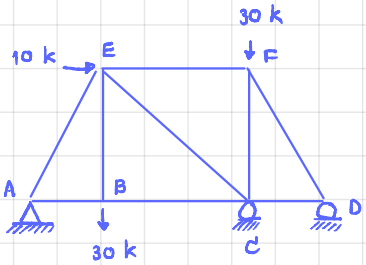
(a)

$D.I. = (m+r)-2j$

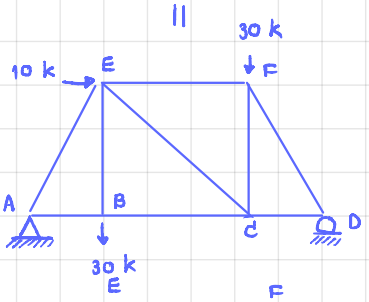
$= (9+4)-2(6) = 1$

เลือก C_y เป็น Redundant

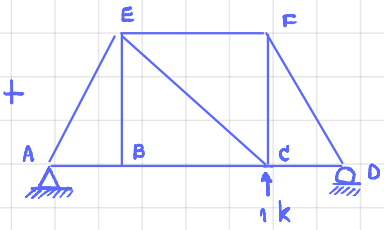
จะได้

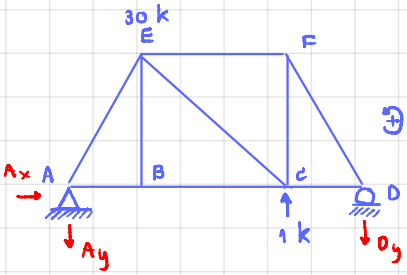


=



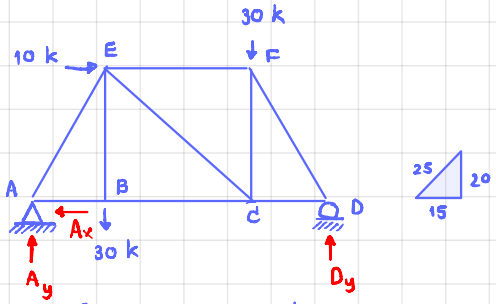
+





จะได้ $\Delta_C = 0 = \Delta_{Cyo} + f_{CCy} C_y$

1) หา Δ_{Cyo} และ f_{CCy}



$[\Sigma F_x = 0]$; $A_x = 10$ k

$[\Sigma M_A = 0]$ ↻ ; $-10(20)-30(30)+D_y(45)-30(15)=0$

$D_y = 34.44$ k

$[\Sigma F_y = 0]$; $A_y = 30+30-34.44 = 25.56$ k

ที่ Joint A ;

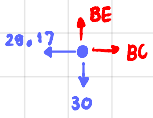
$\Sigma F_y = 0$; $AE \cdot \frac{20}{25} = 25.56$

$AE = 31.95$ k (C)

$\Sigma F_x = 0$; $AB = 31.95 \cdot \frac{15}{25} + 10$

$AB = 29.17$ k (T)

ที่ Joint B ;



BE = 30 k (T)

BC = 29.17 k (T)

ที่ Joint D ;

$[\Sigma F_y = 0]$; $DF \cdot \frac{20}{25} = 34.44$

$DF = 43.05$ k (C)

$[\Sigma F_x = 0]$; $DC = 43.05 \cdot \frac{15}{25}$

$DC = 25.83$ k (T)

ที่ Joint F ;

$[\Sigma F_x = 0]$; $FE = 43.05 \cdot \frac{15}{25}$

$FE = 25.83$ k (C)

$[\Sigma F_y = 0]$; $FC = 43.05 \cdot \frac{20}{25} - 30$

$FC = 4.44$ k (T)

ที่ Joint C ;

$[\Sigma F_y = 0]$; $CE \cdot \frac{20}{25} = 4.44$

$CE = 5.55$ k (C)

ที่ Joint A ;

$[\Sigma F_y = 0]$; $AE \cdot \frac{20}{25} = 0.33$

$AE = 0.4125$ k (T)

$[\Sigma F_x = 0]$; $AB = 0.4125 \cdot \frac{15}{25}$

$AB = 0.2475$ k (C)

ที่ Joint B ;

$BE = 0$

$BC = 0.2475$ k (C)

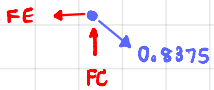
ที่ Joint D ;

$[\Sigma F_y = 0]$; $DF \cdot \frac{20}{25} = 0.67$

$DF = 0.8375$ k (T)

$[\Sigma F_x = 0]$; $DC = 0.8375 \cdot \frac{15}{25} = 0.5025$ k (C)

ที่ Joint F ;



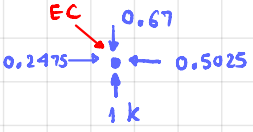
$[\Sigma F_x = 0]$; $FE = 0.8375 \cdot \frac{15}{25}$

$FE = 0.5025$ k (T)

$[\Sigma F_y = 0]$; $FC = 0.8375 \cdot \frac{20}{25}$

$FC = 0.67$ k (C)

ที่ Joint C ;



$[\Sigma F_y = 0]$;

$EC \cdot \frac{20}{25} = 1 - 0.67$

$EC = 0.425$ k (C)

Member	L (ft)	F (k)	F_v (k)	FF_vL (k·ft)	F_v^2L (ft)	$F = F + F_vC_y$
AB	15	29.17	-0.2475	-108.294	0.919	18.9
AE	25	-31.95	0.4125	-329.484	4.254	-14.9
BC	15	29.17	-0.2475	-108.294	0.919	18.9
BE	20	30	0	0	0	30
CD	15	25.83	-0.5025	-194.694	3.788	5.1
CE	25	-5.55	-0.425	58.969	4.5156	-22.8
CF	20	4.44	-0.67	-59.496	8.978	-23.2
DF	25	-43.05	0.8375	-901.359	17.535	-8.5
EF	15	-25.83	0.5025	-194.694	3.788	5.1
			Σ	-1837.346	44.697	

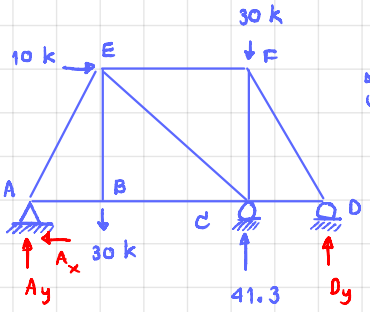
$$\Delta_{co} = \frac{1}{AE}\sum FF_vL = \frac{-1837.3}{AE}$$

$$f_{ccy} = \frac{1}{AE}\sum F_v^2L = \frac{44.7}{AE}$$

∴ จาก $0 = \Delta_{cyo} + f_{ccy}C_y$

$$0 = \frac{-1837.3}{AE} + \frac{44.7}{AE}C_y$$

$$C_y = 41.3 \text{ k}$$



$[\Sigma F_x = 0]$; $A_x = 10$ k

$\circlearrowleft^{+}$ $[\Sigma M_A = 0]$; $-10(20) - 30(15) - 30(30) + 41.3(30) + D_y(45) = 0$

$D_y = 6.9$ k

$[\Sigma F_y = 0]$; $A_y = 60 - 41.3 - 6.9 = 11.8$ k

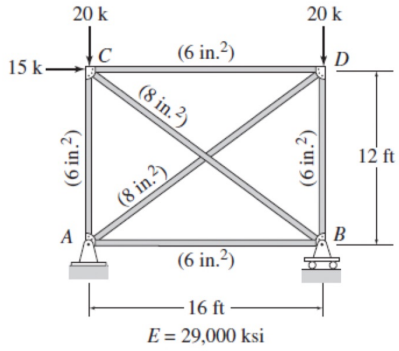


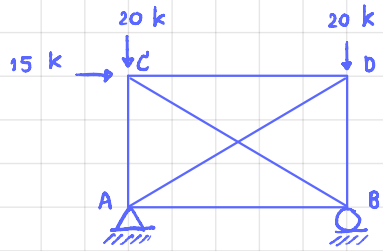
FIG. P13.34

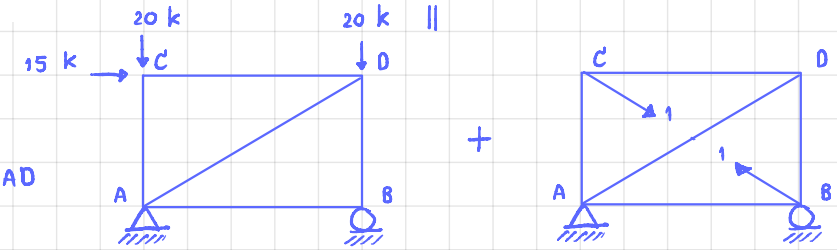
(b)

D.I. = (m+r) − 2j

= (6+3) − 2(4) = 1

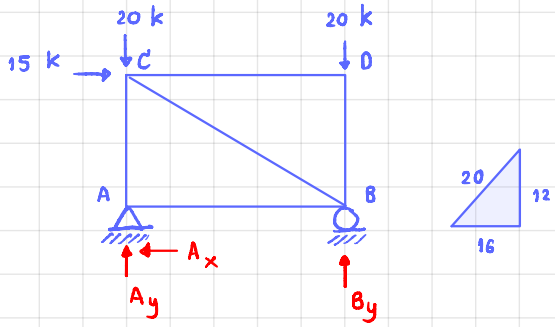
เลือก AD เป็น Redundant





จะได้ $\Delta_{AD} = 0 = \Delta_{AD0} + f_{AD,AD} F_{AD}$

1) หา Δ_{CB0} , $f_{CB,CB}$



$[\Sigma F_x = 0]$; $A_x = 15$ k

$\oplus$ $[\Sigma M_A = 0]$; $B_y(16) - 20(16) - 15(12) = 0$

$B_y = 31.25$ k

$[\Sigma F_y = 0]$; $A_y = 40 - 31.25 = 8.75$ k

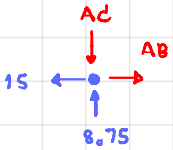
ที่ Joint B ;

$BD = 31.25 - 18.75 \cdot \frac{12}{20} = 20$ k (C)

$[\Sigma F_x = 0]$: $BC \cdot \frac{16}{20} = 15$

$BC = 18.75$ k (C)

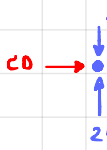
ที่ Joint A ;



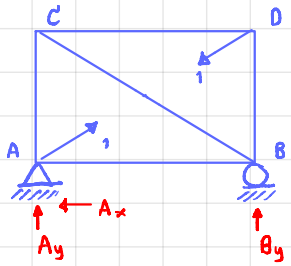
$[\Sigma F_x = 0]$; $AB = 15$ k (T)

$[\Sigma F_y = 0]$; $AC = 8.75$ k (C)

ที่ Joint D ;



$[\Sigma F_x = 0]$; $CD = 0$ k



$[\Sigma F_x = 0]$; $A_x = 0$

ไม่มีแรงภายนอกมากระทำเลย

$[\Sigma F_y = 0]$; จะได้ $A_y = B_y = 0$

ที่ Joint A ;

$[\Sigma F_y = 0]$;

$AC = 1 \cdot \frac{12}{20} = 0.6$ k (C)

$[\Sigma F_x = 0]$;

$AB = 1 \cdot \frac{16}{20} = 0.8$ k (C)

ที่ Joint B ;

$[\Sigma F_x = 0]$;

$BC \cdot \frac{16}{20} = 0.8$

$BC = 1$ k (T)

$[\Sigma F_y = 0]$;

$BD = 1 \times \frac{12}{20} = 0.6$ k (C)

$CD = BA = 0.8$ k (C)

Member	L (ft)	A (in^2)	F (k)	F_v (k)	FF_vL/A $(k\cdot ft)/in^2$	F_v^2L/A $(ft)/in^2$	$F = F + F_v F_{AD}$
AB	16	6	15	-0.8	-32	1.71	11.4
AC	12	6	-8.75	-0.6	10.5	0.72	-11.45
AD	20	8	0	1	0	2.5	4.5
BC	20	8	-18.75	1	-46.875	2.5	-14.25
BD	12	6	-20	-0.6	24	0.72	-22.7
CD	16	6	0	-0.8	0	1.71	3.6
				Σ	44.375	9.86	

$$\Delta_{ADO} = \frac{1}{E}\sum\frac{FF_vL}{A} = -\frac{44.375}{E}$$

$$f_{AD,AD} = \frac{1}{E}\sum\frac{F_v^2L}{A} = \frac{9.86}{E}$$

จาก $0 = \Delta_{ADO} + f_{AD,AD} F_{AD}$

จะได้ $0 = -\frac{44.375}{E} + \frac{9.86}{E} F_{AD}$

$F_{AD} = 4.5\ k$